Factor Investing, Learning from Prices, and Endogenous Uncertainty in Asset Markets

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Motivation and Research Questions

- Knowledge of fundamental value is important for stock investments
- However, since 50s we actively work with factor models
  - Fundamentals are replaced by factors + exposure
  - Research is moving towards the factor-based paradigm
  - "59% of factor investors plan to increase allocations to factor investing ... over the next three years" [INVESCO Global Factor Investing Study (2019)]
- What information should investors pay attention to?
- Incentives to learn about systematic vs. firm-level information?
- What are the equilibrium implications of these information choices?
- How to measure these effects in the data?

This paper

Multi-asset noisy REE with endogenous information choices

- Asset payoffs obey a factor structure
- Investors allocate attention to individual securities and a risk factor

Take the model's predictions to the data for testing

Preview of Results: Theory

Strategic complementarity in investor attention allocation

- More attention to systematic risk can increase systematic uncertainty...
  ... thereby fueling further attention to systematic risk

The mechanism

- Factor structure + risk aversion + limited attention
- As investors shift attention away from individual securities...
  ...these securities' prices become less informative about the risk factor

Multiple regimes of systematic uncertainty and attention allocation

Preview of Results: Empirics

- Estimate a model-based, forward-looking measure for investor attention to systematic information relative to firm-level information (RELUNC)

- Test empirical model predictions
  1. RELUNC follows a regime switching process
  2. High RELUNC regime $\rightarrow$ lower dispersion of market betas
  3. High RELUNC regime $\rightarrow$ lower price sensitivity to firm specific shocks

- Analyze model implications for the high RELUNC regime:
  1. Investment managers face higher risk
  2. Price sensitivity to idiosyncratic earning shocks decreases

Setup: I assets

- $N+1$ short-lived risky assets indexed by $j \in J = \{1, ..., N, f\}$
  - N "individual securities" with payoffs

$$\nu^j = \mu^j + b^j f + \varepsilon^j, \text{ for } j = 1, ..., N$$

  - f is a systematic risk factor
  - $\varepsilon^j \sim N\left(0, 1/\tau_\varepsilon^j\right)$ are residuals uncorrelated with each other and with f
  - An asset with payoff ν^f perfectly correlated with f

$$\nu^f = \mu^f + f$$

- A riskless asset in infinite supply

Setup: II market participants

- Continuum of rational investors
  - CARA utility, risk tolerance ρ
  - Live one period
  - First acquire information about asset payoffs, then trade
- Liquidity trading, independent of fundamentals
  - $z^j \sim N\left(0, 1/\tau_z^j\right)$ is liquidity demand for asset $j \in J$
  - uncorrelated across assets
  - prevent fully revealing prices

Setup: III information acquisition

Investor i allocates fixed resources K to learn signals of the form

$$S_i^j = \nu^j + \xi_i^j, \quad \text{for } j \in J$$

where

- ξ_i^j's are independent of each other for all i and, for some $\omega^j, k_i^j > 0$,

$$\xi_i^j \sim N\left(0, \omega^j / k_i^j\right)$$

- k_i^j: resources investor i allocates to information about asset $j \in J$, s.t.

$$\sum_{j \in J} k_i^j \leq K$$

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Focus on equilibria in which *all investors make the same information choice*

Public belief dynamics

- Systematic risk factor f follows the autoregressive process

$$f' = \phi f + \sigma e'$$

  - $e \sim N(0, 1)$ is an innovation that is independent across periods
  - σ is the volatility process, i.i.d. over time

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- $\mathcal{F}_u$ is beginning-of-period public information set
  - $\mathcal{F}_u$ includes the history of past prices and volatility shocks
- f is unobserved; investors form beliefs about f conditional on $\mathcal{F}_u$

$$f | \mathcal{F}_u \sim N(m, s)$$

- s is "systematic uncertainty"
- The evolution of (m, s) creates a linkage across periods

Equilibrium information choice

- Marginal benefit of allocating resources is equalized across assets

$$Var\left(\nu^j \middle| \mathcal{F}_i\right)/\omega^j \leq \lambda \text{ for all } j \in J$$

with equality iff $k^j > 0$
($\mathcal{F}_i$ is investor i's information set at the trading stage)

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Homogeneous assets

- Assume the N individual securities are homogeneous:
  ($b^j = b$, $\omega^j = \omega$, $\tau_\varepsilon^j = \tau_\varepsilon$, and $\tau_z^j = N\hat{\tau}_z$ for all $j = 1, ..., N$)
- Investors allocate a fraction α of resources K to learn about the factor and $1 - \alpha$ to learn about the N individual securities
- For large N, we can solve for equilibrium in closed form

Proposition

Assume $1 \geq \frac{\omega^f}{\omega} b^2 \left(1 + \frac{2K\rho^2 \hat{\tau}_z}{\omega}\right)$. *There exists* $\bar{N}$ *such that for all* $N > \bar{N}$ *there is a unique equilibrium. As* $N \uparrow \infty$*:*
(i) the attention allocation to the systematic risk factor satisfies

$$\alpha(s) = \begin{cases} 1 & \text{for } s \geq \bar{s} \\ \tilde{\alpha}(s) & \text{for } s \in (\underline{s}, \bar{s}) \\ 0 & \text{for } s \leq \underline{s} \end{cases}$$

for a strictly increasing function $\tilde{\alpha}(s)$ *and thresholds* $\underline{s}, \bar{s}$*;*
(ii) the law of motion for systematic uncertainty satisfies

$$s' = \frac{\phi^2}{s^{-1} + \mathcal{T}_p(s)} + \sigma^2$$

where $\mathcal{T}_p$ *is the precision of the information about* f *that is revealed by prices*

Strategic complementarity in attention allocation

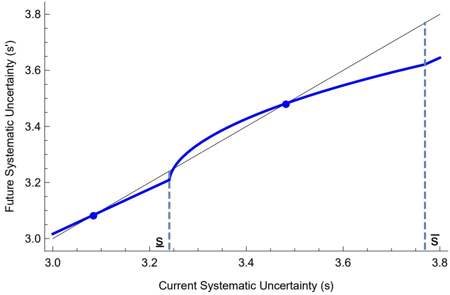
- $\mathcal{T}_p$ measures how much investors learn about f from *all* asset prices

$$\mathcal{T}_p(s) = (\rho K)^2 \left[\left(\frac{\alpha(s)}{\omega^f} \right)^2 \tau_z^f + b^2 \left(\frac{1-\alpha(s)}{\omega} \right)^2 \hat{\tau}_z \right]$$

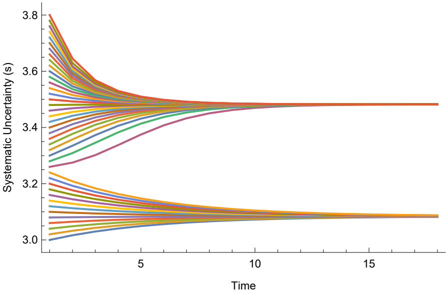
- $\mathcal{T}_p$ decreases in α for α sufficiently small
  - Price informativeness depends on how information is *used*...
    ...and investors trade more aggressively on more precise information

Intertemporal Learning Complementarity: $\alpha \uparrow \Rightarrow \mathcal{T}_p \downarrow \Rightarrow s' \uparrow \Rightarrow \alpha' \uparrow ..$

Uncertainty regimes

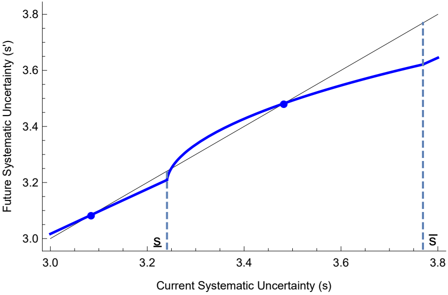


(a) $s' = \frac{\phi^2}{s^{-1} + \mathcal{T}_p(s)} + \sigma^2$



(b) Evolutionary paths of s

Uncertainty regimes



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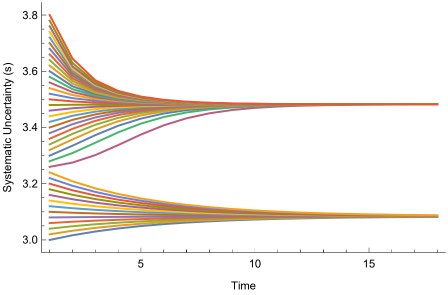

(b) Evolutionary paths of s

Proposition

Under certain conditions, the unique equilibrium has two stable points. The stable point with higher allocation of attention to the risk factor is associated with a higher level of systematic uncertainty.

Heterogeneous assets

Neither asset homogeneity nor an infinite number of assets are necessary for our main results. A numerical example:

(a): $N = \infty$, homogeneous	Low Regime	High Regime
α	0.000	0.693
s	3.084	3.482
(b): $N = 500$, homogeneous	Low Regime	High Regime
α	0.000	0.694
s	3.084	3.481
(c): $N = 500$, heterogeneous	Low Regime	High Regime
α	0.000	0.579
β	0.536	0.312
s	3.235	3.476

Table: In Panel (c) we assume there are 250 assets of type a with parameters $b^a = 1.05, \tau_\varepsilon^a = 0.95, \omega^a = 0.945, \tau_z^a = 0.5$, and 250 assets of type b with parameters $b^b = 0.95, \tau_\varepsilon^b = 1.05, \omega^b = 1.055, \tau_z^b = 2.5$. β denotes the allocation of attention to type a assets (the allocation to type b assets equals $1 - \alpha - \beta$).

Empirical Approach

Model-based uncertainty measure that predicts investors' information choice

- The f.o.c.'s of the model imply for each stock j:

$$Var\left(\nu^j \middle| \mathcal{F}_i\right) \frac{\omega^f}{\omega^j} = Var\left(\nu^f \middle| \mathcal{F}_i\right)$$

- A relative uncertainty measure (RELUNC) minimizes average distance between LHS and RHS and is computed as

$$\text{RELUNC} = \arg\min_{\zeta} \sum_{j=1,\ldots,N} \left(\zeta \times Var\left(\nu^j \middle| \mathcal{F}_i\right) - Var\left(\nu^f \middle| \mathcal{F}_i\right)\right)^2,$$

where under the risk-neutral probability Q [Lemma],

$$Var^Q(\nu) = Var\left(\nu \middle| \mathcal{F}_i\right).$$

Empirical Predictions

1. RELUNC follows a regime switching process with persistent regimes

2. The regime with higher allocation of attention to the risk factor is associated with a lower cross sectional dispersion of risk-neutral betas

3. The regime with higher allocation of attention to the risk factor is associated with lower stock price sensitivity to firm-specific shocks, κ

$$p^j = \overline{p} + (b^j - \overline{b})f + \kappa\left(\varepsilon^j - \overline{\varepsilon}\right) - \lambda(\beta^j - \overline{\beta}) + \eta^j$$

Data for Empirical Analysis

- S&P500 non-financial companies from 1975 to 2019
- Use all companies at some point in the index (684 on average)
- Monthly stock returns and prices from CRSP
- Quarterly fundamentals from Compustat:
  - EBIT scaled by Assets
  - $+$ controls/industry codes, etc.
- Option-based variables from OptionMetrics from 1996 to 2019:
  - Implied volatility (for maturity $+$ delta points) from SurfaceFile
  - Expected variance as simple variance swap rate (Martin, 2017)
- Uncertainty controls:

EPU: US Economic Policy Uncertainty Index, Baker, Bloom, and Davis (2016)
SENT: Investor sentiment, Baker and Wurgler (2006)
TUNC: Average individual implied variances

Relative Uncertainty and Regimes

- Estimate RELUNC from options with 1-year maturity
- Use first differences in RELUNC for parameter estimation
- Use Likelihood Ratio test for regime switching (Di Sanzo, 2009)
- Select Markov-Switching model using information criteria

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- Autoregressive Markov-Switching parameters of RELUNC

Regime	Mean	AR(1)	Volatility	$p_{R->R0}$	Exp. Duration
R0	0.0021	0.1088	0.0003	0.9370	15.9
	(1.54)	(1.53)	(7.50)	(35.11)	
R1	-0.0065	-0.4521	0.0023	0.2842	3.5
	(-1.14)	(-2.86)	(3.72)	(2.84)	

- R0: lower volatility, higher persistence, and lower realized RELUNC
- R0 is the "low RELUNC" regime with low attention to systematic factor

Dispersion of Beta I

- Time-series regression:

$$\sigma(\beta)_t = \alpha + \gamma \boldsymbol{Z_t} + \boldsymbol{\eta X_t} + \varepsilon_t$$

  - $\sigma(\beta)_t$ is the month t cross-sectional standard deviation of market betas
  - $\boldsymbol{Z_t}$ is one of the proxy for the state of relative uncertainty measure
  - $\boldsymbol{X_t}$ is vector of controls

Dispersion of Beta II

	Forward-looking			Historical		
	(1)	(2)	(3)	(4)	(5)	(6)
Constant	...	...	...	...	...	...
RELUNC	-0.522	–	–	-0.258	–	–
	(-10.43)	–	–	(-3.86)	–	–
1_H	–	-0.238	–	–	-0.198	–
	–	(-2.02)	–	–	(-0.78)	–
$P(S_t = H)$	–	–	-0.122	–	–	-0.152
	–	–	(-2.77)	–	–	(-2.29)
Controls	...	...	...	...	...	...
R^2 (%)	75.78	60.26	61.03	20.77	16.73	18.54

- Stock systematic exposure/betas cluster in high RELUNC states

Stock price sensitivity to firm-specific shocks I

- Estimate stock price sensitivity to firm-specific shocks using four-quarter pooled cross-sectional regression:

$$\log\left(\frac{M_t}{A_t}\right)^j = \theta_0 + \theta_1\left(\frac{E_t}{A_t}\right)^j + \theta_2\left(\frac{E_{S,t+h}}{A_{t+h}}\right)^j + \kappa\left(\frac{E_{I,t+h}}{A_{t+h}}\right)^j + \boldsymbol{\eta X_t} + \eta_t^j$$

  - $\left(\frac{E_{S,t+h}}{A_{t+h}}\right)^j$ and $\left(\frac{E_{I,t+h}}{A_{t+h}}\right)^j$ are average future systematic and idiosyncratic earnings over horizon $t+1$ to $t+h$
  - Systematic earnings: fitted values from time series regression of earnings on five principal components; idiosyncratic earnings is the residual
  - $\boldsymbol{X_t}$ captures controls: single industry SIC identifier and market betas

Stock price sensitivity to firm-specific shocks II

- Finally, time-series regression:

$$\kappa_t = \alpha + \gamma \boldsymbol{Z_t} + \boldsymbol{\eta X_t} + \varepsilon_t,$$

  - $\boldsymbol{Z_t}$ is the proxy for the state of relative uncertainty measure computed as the average monthly values for the quarters $t-3$ to t
  - $\boldsymbol{X_t}$ is vector of controls

Stock price sensitivity to firm-specific shocks III

	h = 1			h = 4			h = 12		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	...	...	...	...	...	...	...	...	...
RELUNC	-0.292	–	–	-0.217	–	–	-0.494	–	–
	(-1.59)	–	–	(-1.19)	–	–	(-2.20)	–	–
1_H	–	-0.452	–	–	-0.469	–	–	-0.178	–
	–	(-3.13)	–	–	(-4.28)	–	–	(-1.20)	–
$P(S_t = H)$	–	–	-0.494	–	–	-0.486	–	–	-0.184
	–	–	(-3.77)	–	–	(-4.53)	–	–	(-1.29)
Controls	...	...	...	...	...	...	...	...	...
$R^2(\%)$	19.89	33.85	37.25	36.65	54.12	55.44	37.56	27.42	27.59

- Stock price sensitivity is lower in high RELUNC regime
- The effects are weaker for long-term fundamentals

Economic Implications: Riskiness of Managed Portfolios I

- Tighter clustering of systematic betas in high RELUNC regime suggests higher riskiness of managed investments
- Create six "optimal" strategies with monthly rebalancing:
  - Equal-weighted, unrestricted minimum-variance, constrained minimum-variance, constrained minimum beta, constrained maximum Sharpe ratio, and constrained most diversified portfolios
- ...using a number of standard portfolios:
  - Two-way sorts by eight different characteristics; industry portfolios.
- Each month, compute two risk measures for the next month:
  - Realized volatility, $\sigma_{s,p,t+1}$
  - Realized "Beta Distance", $(1 - \beta_{s,p,t+1})^2$

Economic Implications: Riskiness of Managed Portfolios II

- Finally, run a panel regression with strategy fixed effect:

$$\text{Risk Measure}_{s,p,t+1} = \alpha_{0,s} + \gamma Z_t + \boldsymbol{\nu X_t} + \varepsilon_{t+1},$$

  - $\text{Risk Measure}_{s,p,t+1}$ is either volatility or the “Beta Distance”
  - Z_t is one of the proxies for high relative uncertainty
  - $\boldsymbol{X_t}$ is the vector of controls.

Economic Implications: Riskiness of Managed Portfolios III

	Portfolio Volatility			Beta Distance		
	(1)	(2)	(3)	(4)	(5)	(6)
Constant	...	...	...	...	...	...
RELUNC	0.033	–	–	-0.021	–	–
	(4.18)	–	–	(-2.93)	–	–
1_H	–	0.026	–	–	-0.037	–
	–	(1.74)	–	–	(-2.06)	–
$P(S_t = H)$	–	–	0.031	–	–	-0.021
	–	–	(6.47)	–	–	(-3.44)
Controls	...	...	...	...	...	...
R^2 (%)	45.68	45.62	45.70	16.45	16.43	16.46

- In high RELUNC states, for six strategies across eight data sets:
  - Portfolio volatility is increasing
  - Systematic risk (clustering of market exposure) is increasing

Economic Implications: Reaction To Earnings Surprise I

- Lower price sensitivity to idiosyncratic shocks in high RELUNC states suggests lower reaction of prices to idiosyncratic earning surprises
- Examine the effect of idiosyncratic earning surprise on cumulative abnormal return (CAR) centered on announcement date:
  - Earning surprise (SUE) = difference between actual and consensus estimates
  - Idiosyncratic earning surprise SUE^I is the residual of regressing each firm's SUE on five principal components
  - 3-day CAR centered on announcement date

Economic Implications: Reaction To Earnings Surprise II

- Finally, pooled panel regression:

$$CAR_{i,\tau_t-1\to\tau_t+1} = \alpha_0 + \alpha_{0,Z}Z_t + [\beta_S + \beta_{S,Z}Z_t] \times SUE^S_{i,t} + [\beta_I + \beta_{I,Z}Z_t] \times SUE^I_{i,t} + \boldsymbol{\beta_X X} + \varepsilon_t,$$

  - $CAR_{i,\tau_t-1\to\tau_t+1}$ is the 3-day cumulative abnormal return centered on the announcement date, τ_t,
  - $SUE^{S/I}_{i,t}$ are systematic and idiosyncratic earnings surprise measures
  - Z_t is one of the proxies for high relative uncertainty
  - $\boldsymbol{X}$ is the vector of controls

Economic Implications: Reaction To Earnings Surprise III

	CAR(MKT)			CAR(FF3)		
	(1)	(2)	(3)	(4)	(5)	(6)
SUE_S	0.1529	0.0554	0.0574	0.1521	0.0589	0.0615
	(3.65)	(2.27)	(2.62)	(3.82)	(2.41)	(2.80)
SUE_I	0.1089	0.0776	0.0838	0.1058	0.0785	0.0849
	(2.54)	(3.11)	(3.71)	(2.60)	(3.16)	(3.76)
$SUE_S \times RELUNC$	-0.0307	–	–	-0.0296	–	–
	(-3.30)	–	–	(-3.33)	–	–
$SUE_I \times RELUNC$	-0.0123	–	–	-0.0111	–	–
	(-1.10)	–	–	(-1.02)	–	–
$SUE_S \times 1_H$	–	-0.0296	–	–	-0.0321	–
	–	(-4.22)	–	–	(-4.94)	–
$SUE_I \times 1_H$	–	-0.0449	–	–	-0.0469	–
	–	(-6.28)	–	–	(-6.94)	–
$SUE_S \times P(S_t = H)$	–	–	-0.0264	–	–	-0.0293
	–	–	(-1.85)	–	–	(-2.14)
$SUE_I \times P(S_t = H)$	–	–	-0.0410	–	–	-0.0430
	–	–	(-2.89)	–	–	(-3.14)
R^2 (%)	0.411	0.378	0.373	0.403	0.382	0.376

Conclusion

- Endogenous information model with many risky assets
- Factor structure of asset payoffs

  $\rightarrow$ signals about individual assets are informative about the risk factor
- Focus on systematic sources of risk can increases systematic uncertainty
- Multiple regimes of systematic uncertainty and attention allocation

- High uncertainty regime is associated with
  - Lower dispersion of market betas
  - Lower sensitivity of stock prices to firm-specific shocks

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