

# Why is capital slow moving? Hysteresis in price efficiency and the dynamics of informed capital

James Dow<sup>1</sup> Jungsuk Han<sup>2</sup> Francesco Sangiorgi<sup>3</sup>

<sup>1</sup>London Business School <sup>2</sup>Stockholm School of Economics

<sup>3</sup>Frankfurt School of Finance and Management

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# Imperfect Arbitrage and Slow Moving Capital

- ▶ Privately-informed investors exploit price inefficiencies for profits (Grossman and Stiglitz (1980), Kyle (1985))
  - ▶ With enough capital around, prices converge to fundamental value
  - ▶ Constrained capital may break down such pricing mechanism
- ▶ Slow moving capital: surplus capital should flow from other markets to exploit arbitrage opportunities, but this reallocation happens slowly (Mitchell, Pedersen, and Pulvino (2007) and Duffie (2010))
- ▶ What stops capital from flowing to correct mispricing?

# This paper

Dynamic noisy REE model of informed trading under a capital constraint with endogenous capital allocation across markets

## Active Arbitrage Capital

- ▶ Traditional perspective: amount of arbitrage capital is what matters
- ▶ However, arbitrage capital may be either **active** or **inactive**
  - ▶ Active capital: ready to be deployed for new investment opportunities
  - ▶ Inactive (or “locked in”) capital: already engaged in existing investment
- ▶ Stock vs. flow of arbitrage capital
  - ▶ Arbitrage capital deploys slowly because capital-constrained informed investors hold positions in other markets which they prefer not to unwind (e.g., Von Beschwitz, Lunghi and Schmidt, 2017)

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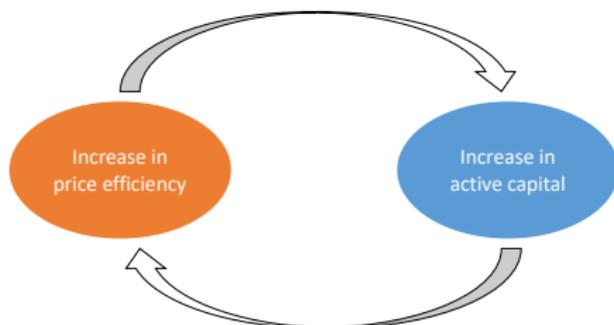
Our new perspective: amount of **active** capital is an important state variable (rather than just amount of capital)

## This paper (cont'd)

Dynamics of active capital provide a mechanism for slow moving capital

### Insights

- ▶ Dual role of price efficiency:
  - ▶ Probability of implementing a new trade successfully
  - ▶ Duration of existing investment (or, acceleration of capital redeployment)
- ▶ Feedback between active capital and price efficiency
  - ▶ Intense arbitrage activity speeds up price efficiency
  - ▶ High price efficiency speeds up the rate at which capital is redeployed



# Preview of Results

- ▶ The feedback gives rise to multiple regimes of price efficiency
- ▶ Level of active capital is the state variable which decides the regime
  - ▶ A threshold level for active capital separates two domains of attraction
- ▶ As active capital evolves over time as a result of shocks, the market undergoes regime shifts on the equilibrium path

# Preview of Results

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- ▶ As active capital evolves over time as a result of shocks, the market undergoes regime shifts on the equilibrium path

## Response to temporary shocks:

- ▶ Slow recovery as arbitrageurs trade off higher mispricing with longer duration investment as active capital recovers
- ▶ Temporary shocks have persistent effects when they push active capital across the threshold and trigger a shift in regime
  - ▶ Flight-to-liquidity: arbitrage capital flows to more liquid investments
  - ▶ Hysteresis: price inefficiency persists even when the initial cause is removed

# Literature

- ▶ **Noisy rational expectations equilibrium model:** Grossman and Stiglitz (1980), Kyle (1985), Dow and Gorton (1994), Wang (1993, 1994), Vives (1995), Yuan (2005), Goldstein, Li, and Yang (2014), Cespa and Foucault (2014), Dow and Han (2018)
  - ▶ Existence of noise combined with constraints or risk aversion of informed investors prevents full revelation of private information
- ▶ **Limits to arbitrage:** Allen and Gale (1994), Shleifer and Vishny (1997), Kyle and Xiong (2001), Gromb and Vayanos (2002), Kondor (2009), Brunnermeier and Pedersen (2009), Gromb and Vayanos (2018), Buss and Dumas (2018), Rostek and Weretka (2015)
  - ▶ Constrained capital of arbitrageurs allows mispricing to persist with potential amplifications

We combine limits to arbitrage with a noisy REE model of asset prices

# Roadmap

1. **Model Setup**
2. Equilibrium
3. Implications
4. Shock responses
5. Conclusion

# Setup

- ▶ Infinite horizon economy in discrete time  $t = 1, 2, \dots$
- ▶ Competitive risk-neutral investors
- ▶ Continuum of risky assets
- ▶ Risk-free asset with exogenous return  $R_f$
- ▶ Discount factor  $\beta = 1/R_f$

# Assets

- ▶ Two markets: “long-term market” (market  $L$ ) and “short-term market” (market  $S$ )
  - ▶ Assets in market  $L$  pay liquidating dividend each period with prob.  $q < 1$
  - ▶ Assets in market  $S$  pay liquidating dividend each period
- ▶ At maturity, asset  $i$  in market  $h$  pays  $v_i$  which is either good  $v_i = V_h^G$  or bad  $v_i = V_h^B$  with equal probabilities
- ▶ At maturity, assets are replaced
- ▶ Payoffs are independent across assets and time

# Arbitrageurs

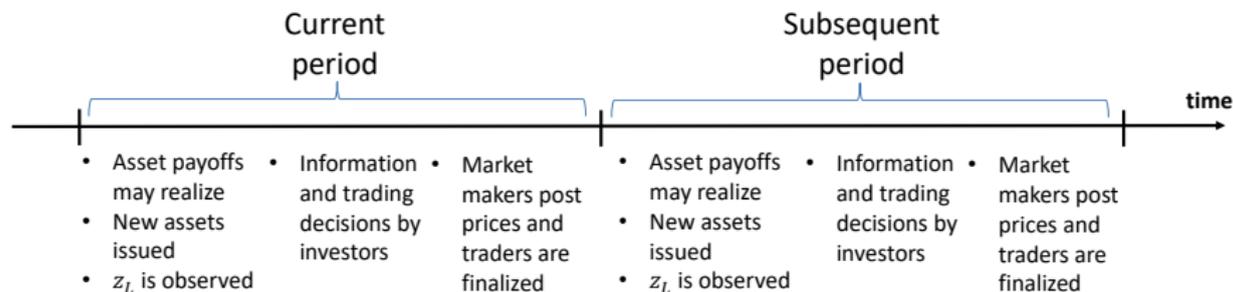
- ▶ Infinitely lived, risk-neutral, competitive
- ▶ Can produce private information on one asset in each period
- ▶ Can freely move across different markets (or asset classes)
- ▶ Capital constrained: can hold one risky position in each period  
(one unit, long or short)

# Market microstructure

## Trading in a similar manner to Kyle model (1985)

- ▶ Informed arbitrageurs
  - ▶ Submit one unit order of long or short position  $x_i^j \in \{-1, 0, 1\}$
- ▶ Noise traders
  - ▶ Submit exogenous order flow  $\zeta_i$  which is i.i.d uniform on  $[-z_S, z_S]$  in market  $S$  and i.i.d uniform on  $[-z_L, z_L]$  in market  $L$
  - ▶  $z_L$  follows a Markov process with  $N$  states
- ▶ Competitive risk-neutral market makers
  - ▶ Semi-strong form efficient prices

# Timeline



# States of arbitrage capital

Each period, arbitrageurs belong to:

- ▶  $\xi$  mass: active arbitrageurs, free for new investment
- ▶  $1 - \xi$  mass: inactive (or “locked-in”) arbitrageurs who hold positions in unrevealed assets (in order to engage in new investment, they must liquidate their positions first)

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Active arbitrageurs choose between market  $L$  and market  $S$

- ▶  $\delta$  is the portion of active arbitrageurs choosing market  $L$  (thus,  $1 - \delta$  portion chooses market  $S$ )

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Inactive arbitrageurs choose between liquidating early or holding on to their position one more period

- ▶  $\eta$  is the portion of inactive arbitrageurs choosing to liquidate early  
(in equilibrium,  $\eta = 0$ )

# Price Efficiency

- ▶ Each active arbitrageur ( $\xi$  mass) chooses market  $h \in \{L, S\}$ , then, collects information on one asset in that market
- ▶ Price either fully revealing or completely uninformative (uniform noise)
  - ▶  $P^G$  ( $P^B$ ) denotes the fully-revealing price for a good (bad) quality asset;  $P^0$  denotes the non-revealing price
- ▶ Price efficiency: probability of revealing fundamental value  $v_i$

$$\lambda_i = \frac{\text{informed order flow in asset } i}{\text{noise trading intensity in asset } i}$$

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- ▶ In market-wise symmetric equilibrium

$$\lambda_L = \frac{\delta\xi}{z_L}; \quad \lambda_S = \frac{(1-\delta)\xi}{z_S}$$

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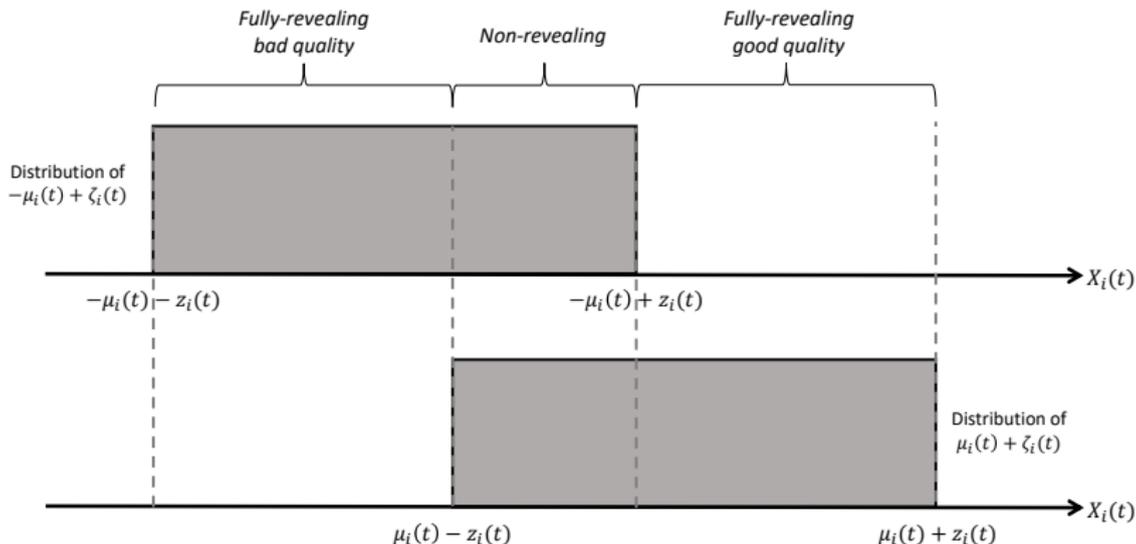
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- ▶  $\lambda_h$  is the “speed of convergence” of prices to fundamentals in market  $h$

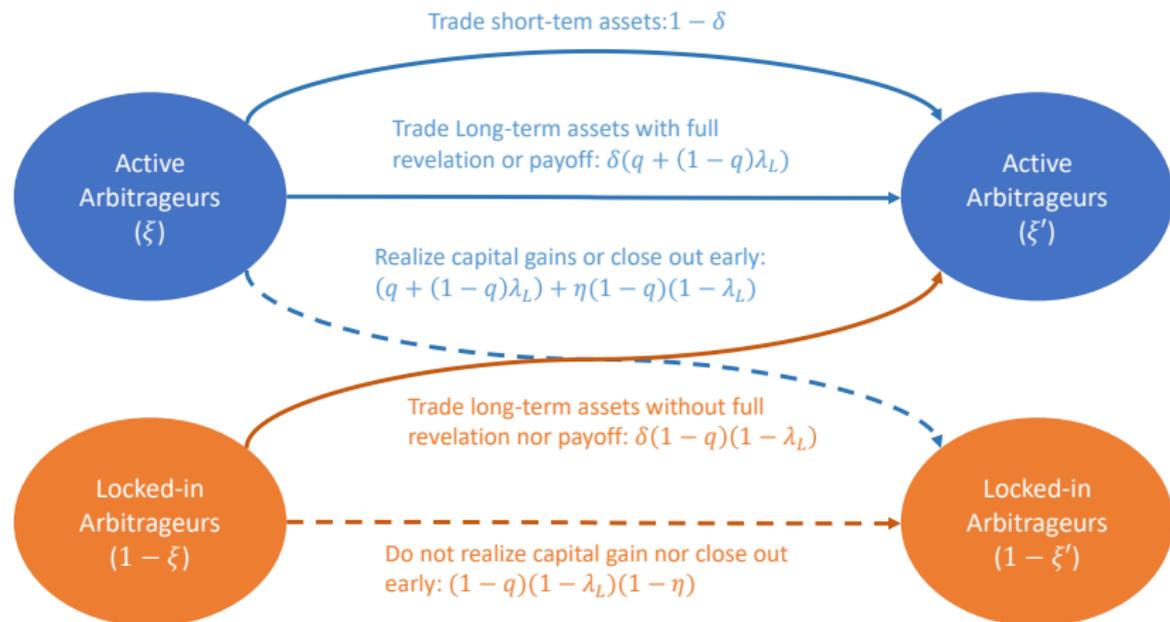
## Illustration of Price Informativeness



Distribution of aggregate order flow  $X_i(t) = \int_{a \in \mathcal{A}} x_i^a(t) da + \zeta_i(t)$  conditional on asset  $i$ 's quality being bad (top panel) or good (bottom panel), and the associated regions of full-revelation and non-revelation.

1. Model Setup
2. **Equilibrium**
3. Implications
4. Shock responses
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# Laws of motion



$$\xi' = (1 - \delta)\xi + (\delta\xi + 1 - \xi)(q + (1 - q)\lambda_L) + (1 - \xi)\eta(1 - q)(1 - \lambda_L)$$

## Portfolio choice

Value function of active arbitrageurs given the state  $\theta = (\xi, z_L)$ :

$$J_f(\theta) = \max(J_L(\theta), J_S(\theta))$$

where the value functions of choosing market  $S$  and  $L$  are, respectively:

$$J_S(\theta) = -(\lambda_S P^G + (1 - \lambda_S) P^0) + \beta \left[ V_S^G + E[J_f(\dot{\theta}) | \theta] \right],$$

$$\begin{aligned} J_L(\theta) = & -(\lambda_L P^G + (1 - \lambda_L) P^0) \\ & + \beta \left[ q V_L^G + (1 - q) \lambda_L P^G + (1 - (1 - \lambda_L)(1 - q)) E[J_f(\dot{\theta}) | \theta] \right. \\ & \left. + (1 - \lambda_L)(1 - q) E[J_L(\theta') | \theta] \right] \end{aligned}$$

In an interior equilibrium, active investors are indifferent between  $S$  and  $L$

$$J_S(\theta) = J_L(\theta)$$

## Portfolio choice (continued)

Value function of locked-in arbitrageurs:

$$J_l(\theta) = \max(J_E(\theta), J_H(\theta))$$

where the value functions of closing out early and staying are, respectively:

$$J_E(\theta) = \lambda_L P^G + (1 - \lambda_L) P^0 + \beta E[J_f(\theta') | \theta]$$

$$J_H(\theta) = \lambda_L P^G + (1 - \lambda_L) P^0 + J_L(\theta)$$

In an interior equilibrium, we show that locked-in arbitrageurs prefer staying

$$J_E(\theta) < J_H(\theta)$$

Thus, there is no early exit ( $\eta = 0$ ) in equilibrium

# Equilibrium

## Definition

A stationary equilibrium is a collection of value functions  $J_f, J_l, J_L, J_S, J_E, J_H$ , capital allocation function  $\delta$ , exit function  $\eta$ , price efficiency measures  $\lambda_L, \lambda_S$ , law of motion for the mass of active arbitrageurs  $\xi$  such that

1.  $J_f, J_l, J_L, J_S, J_E, J_H, \delta, \eta$  satisfy the system of equations described above.
2.  $\lambda_L$  and  $\lambda_S$  correspond to the probability that prices, which are determined by  $P_i = E[\beta^{\tau_i} v_i | \theta, X_i]$ , reveal true asset values in market  $L$  and  $S$ , respectively.
3. The law of motion for  $\xi$  satisfies
$$\xi' = (1 - \delta)\xi + (1 - (1 - \delta)\xi)(q + (1 - q)\lambda_L) + (1 - \xi)\eta(1 - q)(1 - \lambda_L)$$

# Equilibrium

## Proposition

*Under certain conditions, there exists a unique stationary interior equilibrium in which price efficiency in the long-term market ( $\lambda_L$ ) is monotone increasing in active capital ( $\xi$ ). Furthermore, price efficiency  $\lambda_L$  is monotone decreasing in noise trading intensity  $z_L$*

1. Model Setup
2. Equilibrium
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## Steady state analysis

Assume initially that noise trading intensity is at a constant level

Value functions:

$$J_S^* = \frac{(P^G - P^0)(1 - \lambda_S^*)}{1 - \beta}.$$

$$J_L^* = \frac{(P^G - P^0)(1 - \lambda_L^*)[1 - \beta(1 - q)(1 - \lambda_L^*)]}{1 - \beta},$$

Dual role of price efficiency on informed traders' profits

- 1 With higher  $\lambda_L$  (also  $\lambda_S$ ) mispricing wedge is lower and so are trading gains
  - ▶ “Purchase Price Effect”
- 2 With higher  $\lambda_L$ , locked-in arbitrageurs can realize capital gains at a faster rate, thereby becoming active again more quickly
  - ▶ “Ex-ante Unlocking Effect”

## Steady state analysis (cont'd)

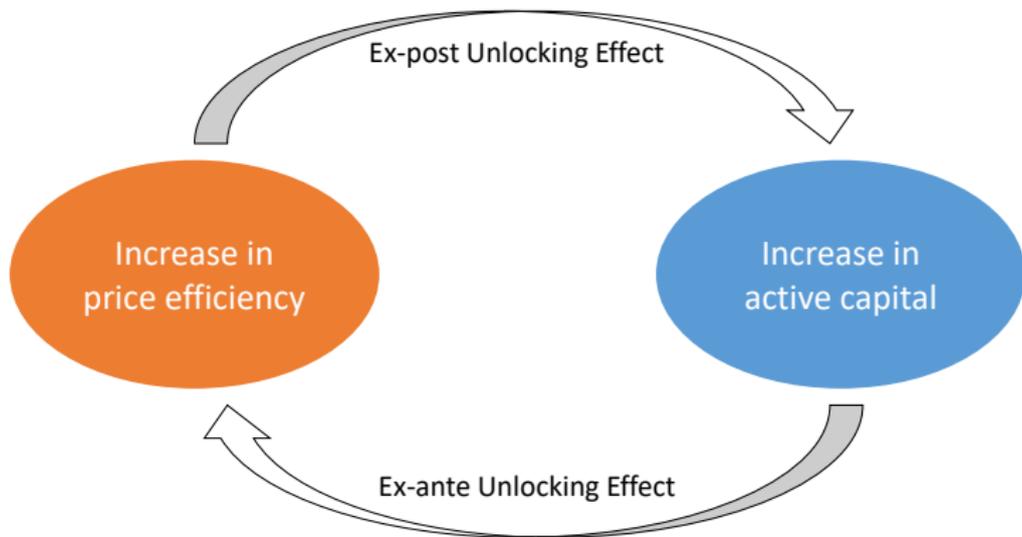
The law of motion for active capital  $\xi$

$$\xi^* = (1 - \delta^*)\xi^* + (\delta^*\xi^* + 1 - \xi^*)(q + (1 - q)\lambda_L^*)$$

- ▶ Higher price efficiency in market  $L$  increases the rate at which arbitrage capital is released from this market
- ▶ This tends to increase  $\xi^*$  (“Ex-post Unlocking Effect”)

# Feedback between Active Capital and Price Efficiency

An increase in price efficiency releases locked-in capital at a faster rate, thereby increasing active capital in the future



An increase in future active capital improves future price efficiency, inducing arbitrageurs to invest more in the long-term market, thereby increasing current price efficiency further

# Steady state equilibrium

Assume initially that noise trading intensity is at a constant level

## Proposition

*(i) A steady state equilibrium exists, and there is either one or two stable steady state equilibria. (ii) There exist thresholds of  $q$  and  $\beta$  which ensure either uniqueness or multiplicity of steady state equilibria.*

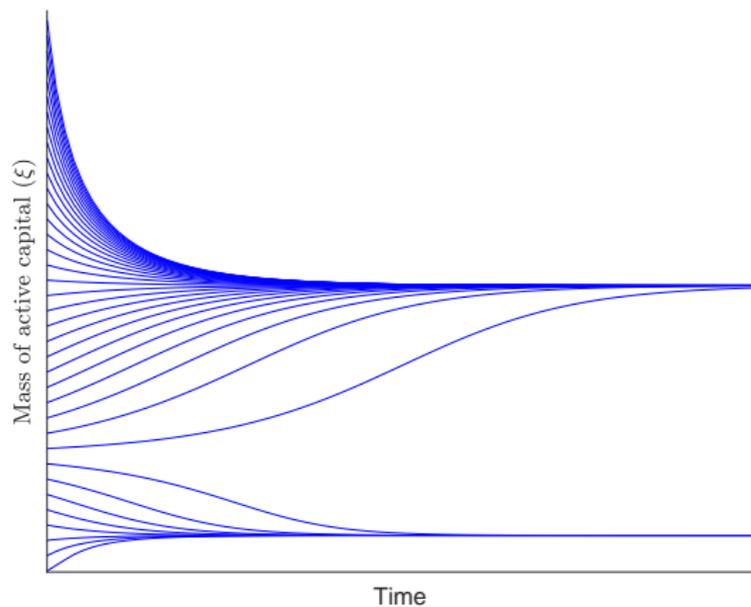
## Corollary

*With multiple steady state equilibria, an equilibrium with larger amount of active capital features higher levels of price efficiency and trading volume in market  $L$  and lower levels of price efficiency and trading volume in market  $S$  compared to an equilibrium with smaller amount of active capital.*

1. Model Setup
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# Evolutionary Paths of the Mass of Active Capital

Two locally stable fixed points for  $z_L = \text{Normal}$



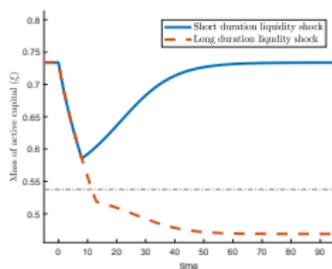
# Numerical Analysis of General Stochastic Model

- ▶ Parameter values:  $q = .01, z_S = .475, \beta = .95$
- ▶ Noise trading intensity  $z_L$  follows a Markov process with 3 states  $z_L \in \{Low(.6), Normal(.65), High(.7)\}$
- ▶ The transition matrix is

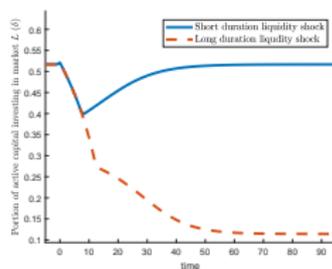
$$\Omega = \begin{bmatrix} 0.46 & 0.54 & 0.00 \\ 0.12 & 0.76 & 0.12 \\ 0.00 & 0.5 & 0.5 \end{bmatrix}$$

- ▶ Normal (i.e., intermediate) noise trading intensity state is persistent; high and low states are transitory
- ▶ We illustrate the effects of a temporary deviation of  $z_L$  from its normal level to the high level

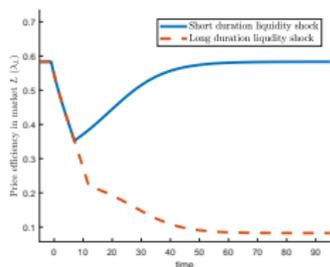
# Transitional dynamics (Different shock durations)



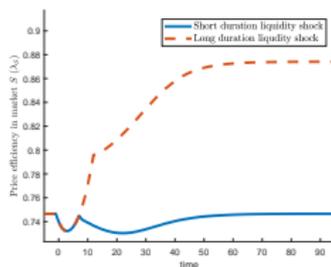
(a) Active arbitrageurs  $\xi$



(b) Investment in  $L$  market  $\delta$



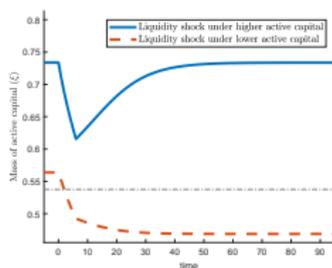
(c) Price informativeness  $\lambda_L$



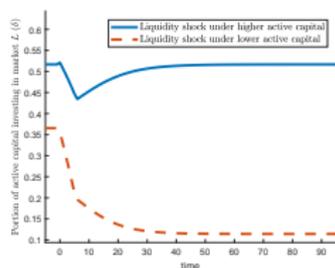
(d) Price informativeness  $\lambda_S$

With longer shock, efficiency does not recover on its own even after the shock is removed

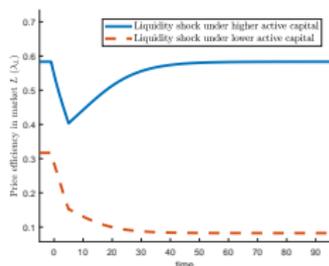
# Transitional dynamics (Different initial conditions)



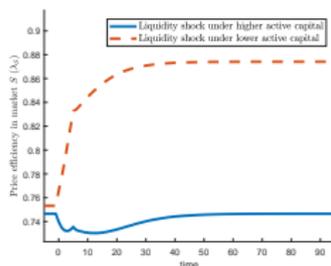
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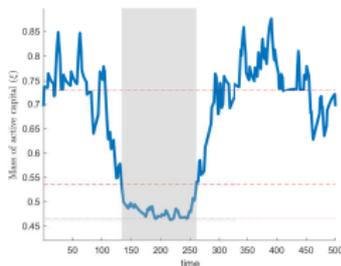
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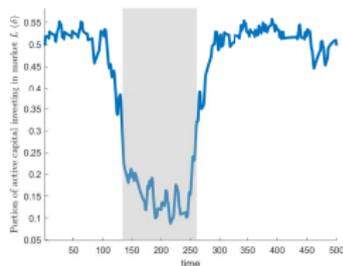
(d) Price informativeness  $\lambda_S$

With initially low active capital, efficiency does not recover on its own even after the shock is removed

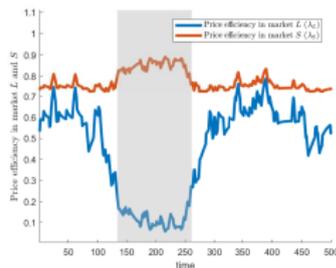
# Simulation: in & out efficiency regimes



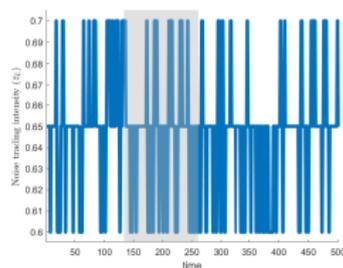
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(c) Price informativeness



(d) noise trading intensity  $z_L$

Persistent endogenous efficiency regimes triggered by temporary shocks

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# Discussion and empirical implications

## Model

- ▶ Cross-sectional difference in liquidity across markets predict future illiquidity as well as slow convergence of price to fundamental
- ▶ Transient shocks which cause limited capital to get redeployed more slowly can cause flight-to-liquidity and liquidity hysteresis
- ▶ Multiple liquidity regimes
- ▶ Capital flows into more illiquid assets as active capital expands

## Evidence

- ▶ Acharya, Amihud, and Bharath (2013): (i) different liquidity regimes for corporate bonds; (ii) flight-to-liquidity in conjunction with the illiquid regime
- ▶ Beber, Brandt, and Kavajecz (2007): capital flows into Euro-area government bonds mostly determined by “chasing liquidity”
- ▶ Liquidity crises and delayed recovery of liquidity in the aftermath (e.g., Mitchell, Pedersen, and Pulvino (2007))
- ▶ Reaching-for-Yield (e.g. Becker and Ivashina (2015))

# Conclusion

We provide a new mechanism for slow moving capital using endogenous dynamics of price efficiency

- ▶ Hysteresis in price inefficiency: price inefficiency persists even when the initial cause is removed because capital becomes trapped in existing investment
- ▶ Trade-off (or dual role) of price efficiency for speculative trading: price purchase effect vs. unlocking effect
- ▶ Slow moving capital/Flight-to-liquidity: arbitrage capital may actually flow out of the long-term market and only come back later
- ▶ Regime shifts in price inefficiency: persistent endogenous inefficiency regimes may be triggered by temporary shocks